

# Çankaya University – ECE Department – ECE 376

Student Name :  
Student Number :

Open source exam  
Duration : 2 hours

## Questions

1. (70 Points) A DS spread spectrum communication system with two users (with two message signals of  $v_1(t)$  and  $v_2(t)$ ) is given. Each user transmits at a bit rate of 4 kbits / sec. For this communication system, to provide a  $P_e$  (probability error) of  $10^{-4}$ , find the SNR (due to interference between users arising from the common use of time and frequency axis) and  $L_c$  (processing gain) and  $T_c$  (chip duration of the PN sequences). State what assumptions you have made in these calculations.

Now assuming that these two users are assigned two of the following four spreading (PN) sequences,

$${}_1C = [-1 \ -1 \ 1 \ -1 \ -1 \ 1 \ -1]$$

$${}_2C = [-1 \ -1 \ 1 \ 1 \ -1 \ -1 \ 1]$$

$${}_3C = [-1 \ -1 \ 1 \ -1 \ 1 \ 1 \ 1]$$

$${}_4C = [-1 \ -1 \ 1 \ 1 \ 1 \ -1 \ 1]$$

Explain which of the  ${}_1C$ ,  ${}_2C$ ,  ${}_3C$  and  ${}_4C$  you would choose as PN sequences of the two users bearing in mind that the time shifted autocorrelation function of the chosen PN sequences should make the chosen PN sequence maximum length. Find the new  $L_c$  and  $T_c$ . Find the time shifted autocorrelation and time shifted crosscorrelation of the chosen PN sequences and plot them. Note that autocorrelations and crosscorrelations can be obtained from matrix multiplication of PN sequences.

For the above DS spread spectrum system, draw transmitter and receiver block diagrams and plot time waveforms and spectrums of signals, labelling these plots with numeric values.

*Solution 1. At  $P_e = 10^{-4} = Q(x)$ , from tables*

$$x \approx 3.7 = \sqrt{\frac{2 E_b}{I_0}}$$

*where  $I_0$  is the interference from other user. Assuming each user arrives at receiver*

With a power level of  $P_b$  and  $T_b$  is the bit duration of each user i.e.  $T_b = 1/4 \times 10^3$  sec.

$$E_b = P_b T_b \quad \text{and} \quad R_b = 4 \text{ kbits/sec} = 1/T_b$$

Due to theoretical arrangement of PN sequences the intended user will be despread while the other user will spread. For other user,  $P_b$  will spread into a frequency band of  $1/T_c$  ( $T_c =$  chip duration of PN sequence), thus

$$I_0 = \frac{P_b}{1/T_c} = \frac{P_b}{R_c} \quad , \quad R_c = \text{Rate of PN sequence (BIT)}$$

$$\frac{E_b}{I_0} = \frac{P_b/T_b}{P_b/(1/T_c)} = \frac{R_c}{R_b} = \frac{T_b}{T_c} = L_c = \frac{(3.7)^2}{2}$$

$$R_c = R_b \frac{(3.7)^2}{2} = 4 \text{ kbits/sec} \times 6.845$$

$$= 27.38 \text{ kbits/sec}$$

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So for the case of a DS-SS spread spectrum system consisting of two users, PN sequence has to be only 6.845 times longer than the bit rate of each message signal, this also means that

$$T_c = \frac{T_b}{6.845} = \frac{0.25 \text{ msec}}{6.845} = 36.5 \mu\text{sec}$$

In the given PN sequences,  $L_c = 7$  which just satisfies the above condition. Now to see which of the  $1^c, 2^c, 3^c, 4^c$  of the PN sequences are suitable, we calculate autocorrelations of all sequences as follows

$$1^c \times 1^{cT} (t - nT_c) \quad \begin{array}{l} T_c \rightarrow \text{transpose} \\ n = 0 \dots 7; \text{cyclic shift} \end{array}$$

$$= [7, -1, -1, 3, 3, -1, -1]$$

$$2^c \times 2^{cT} (t - nT_c)$$

$$= [7, -1, -5, 3, 3, -5, -1]$$

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$${}_3C \times {}_3C^T (t - NT_c)$$

$$= [7, -1, -1, -1, -1, -1, -1]$$

$${}_4C \times {}_4C^T (t - NT_c)$$

$$= [7, -1, -1, -1, -1, -1, -1]$$

So only  ${}_3C$  and  ${}_4C$  have the desired property of maximum length, thus  ${}_3C$  and  ${}_4C$  can be chosen as PN sequences.

The same result could have been obtained by inspecting  ${}_1C$  and  ${}_2C$  as follows

$${}_1C = [-1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1]$$

← same pattern →

$${}_2C = [-1 \quad -1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1]$$

↘ same pattern ↙

whereas in  ${}_3C$  and  ${}_4C$  no such repetitions are encountered.

Cross correlations of max. length sequences

${}_3C$  and  ${}_4C$

$${}_3C * {}_4C(t - nT_c)$$

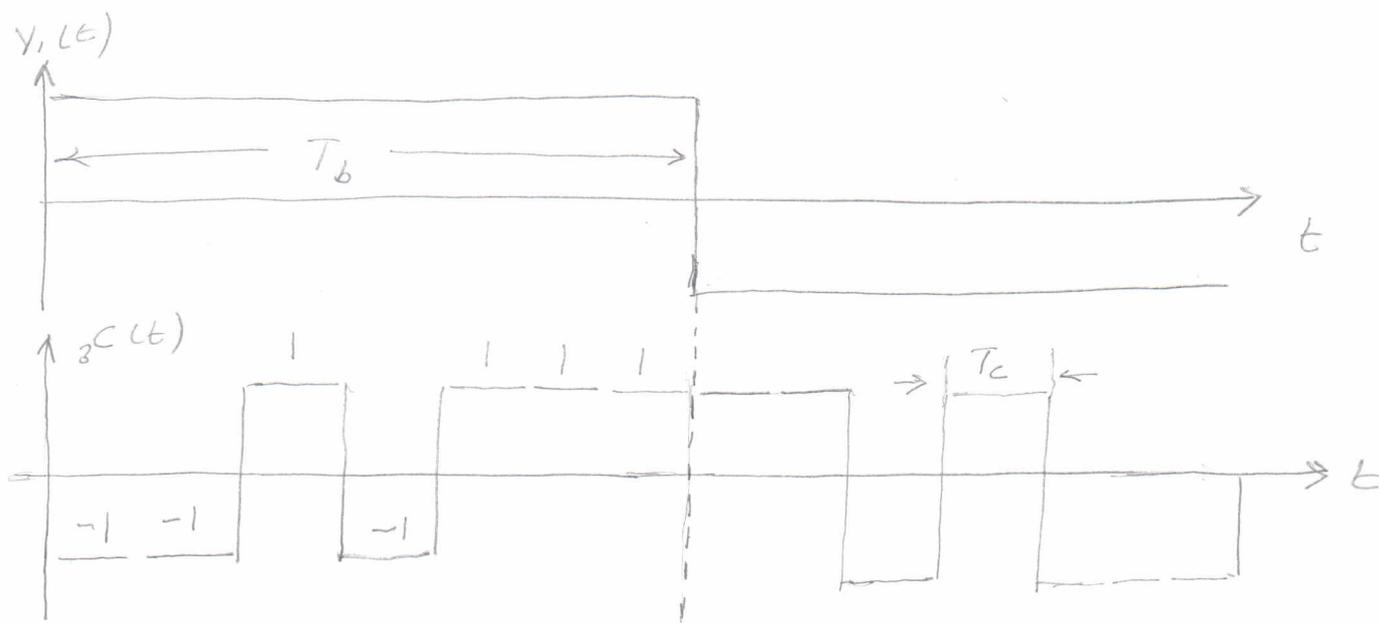
$$= [3, -1, -1, -5, 3, 3, -1]$$

$${}_4C * {}_3C(t - nT_c)$$

$$= [3, -1, 3, 3, -5, -1, -1]$$

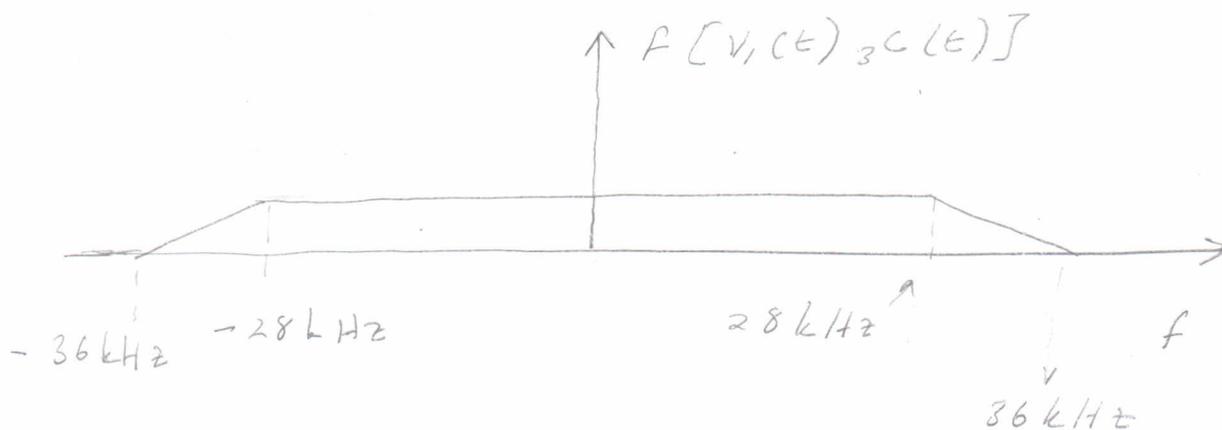
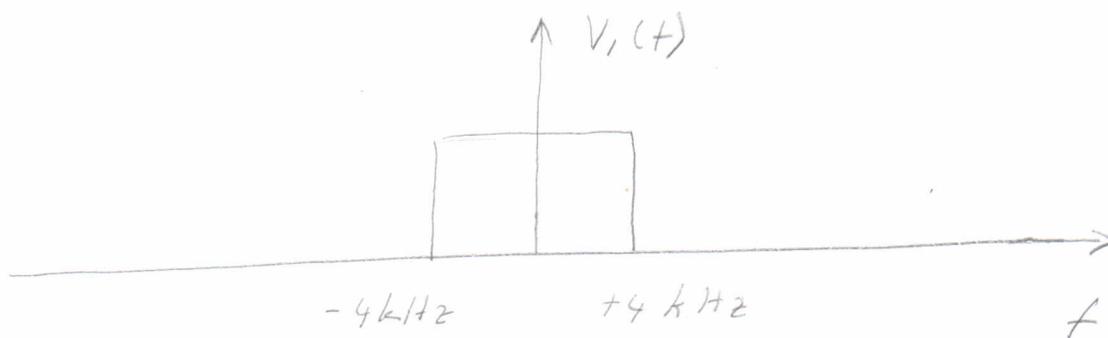
Crosscorrelation properties are not so desirable

Approximate time waveforms for  $v_1(t)$  and  ${}_3C(t)$



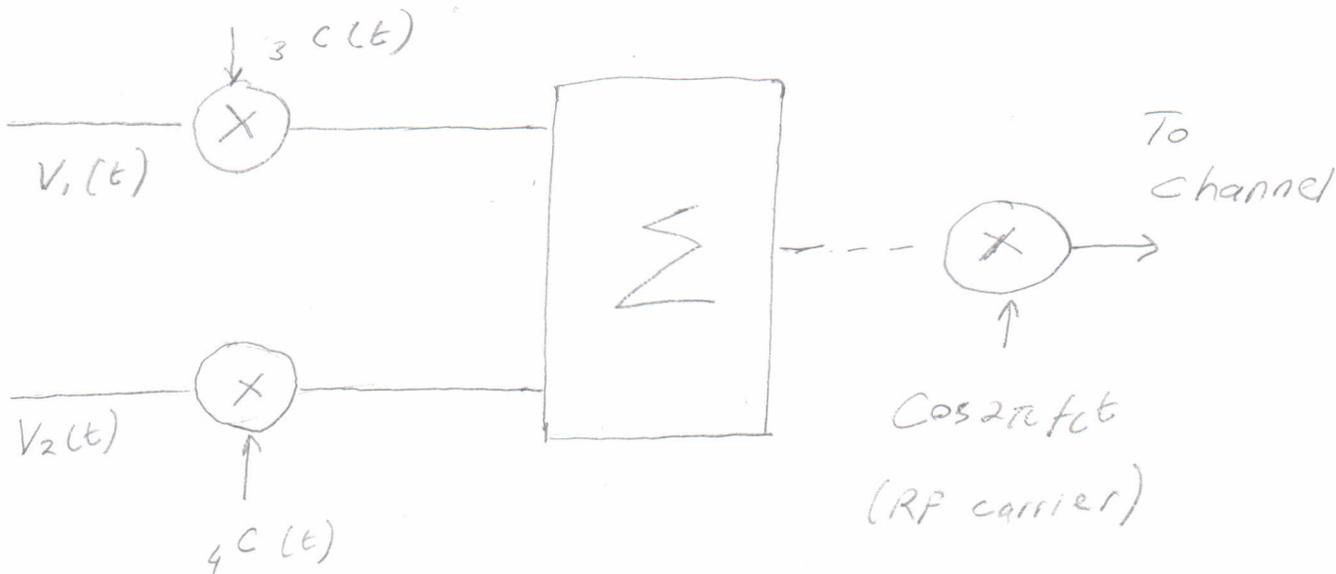
Approximate frequency spectrums of  $V_1(f)$

and  $F[V_1(t) \cdot c(t)]$

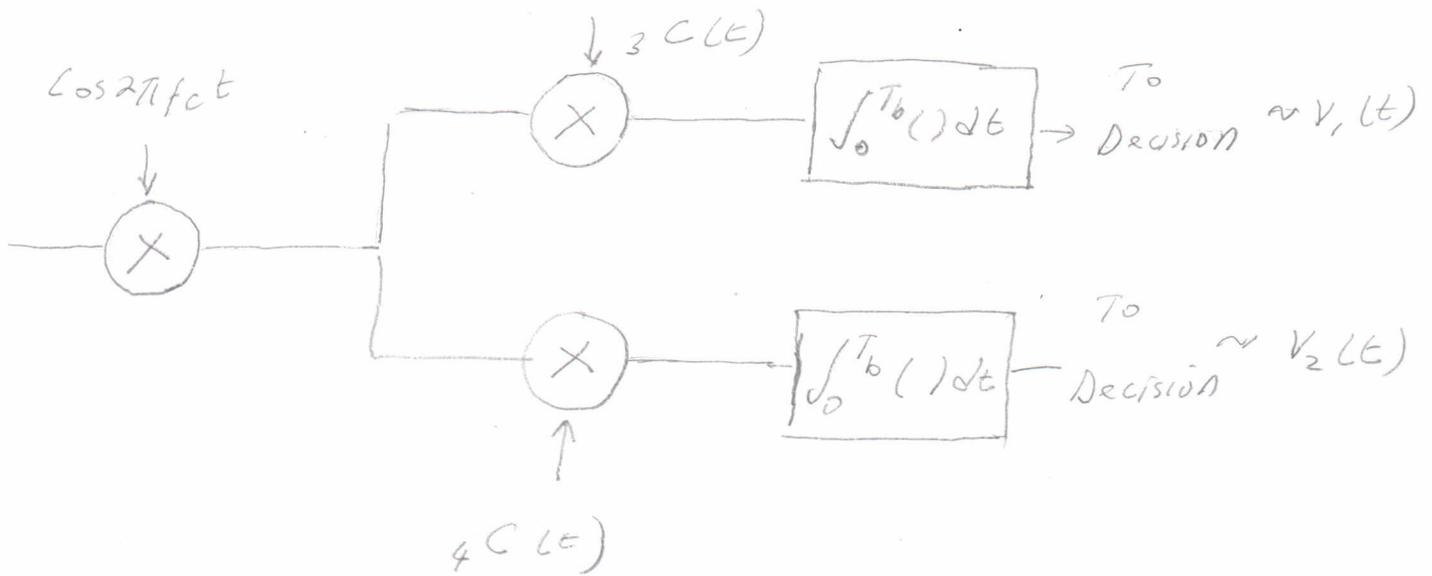


(Two user)

D) Spread Spectrum System Transmitter



## DS Spread Spectrum System Receiver



2. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) In OFDM, subcarriers are determined taking into account properties of noise : *False*  
in OFDM, subcarriers are determined taking into account the orthogonality condition

$$f_i - f_k = \frac{N}{T_s} \text{ or } \frac{N}{T} \text{ where } T = NT_s$$

b) OFDM offers the advantage of transmitting the message signal in a narrower bandwidth : *False*, OFDM is an implementation to split up the channel into narrower bandwidth slices so that each slice appears rectangular. It is not the intention to save on bandwidth in OFDM.

c) Symbols in different time slices (slots) of message signal are assigned to different subcarrier frequencies in OFDM : *True*, by converting QAM or PSK signals from serial to parallel, and assigning each symbol a subcarrier, we effectively assign different time slices of QAM or PSK signal to different

d) In OFDM, we get the message signal from PSK or QAM modulator : *frequencies*  
*True*, input (message) signal to OFDM comes from a QAM or PSK modulator.

e) OFDM is to be used in digital audio broadcasting : *True*, but incomplete  
OFDM will also be used in digital video broadcasting and probably in 4G networks